Gauge Invariance and Localizability in Electromagnetic Theory

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It is pointed out that there is no connection between gauge invariance and localizability in electromagnetic theory such as that discussed by Aharonov and Bohm.

THE relationship between gauge invariance and localizability in electromagnetic theory has been discussed by Aharonov and Bohm.[1] They conclude that abandoning gauge invariance is equivalent to altering some of the assumptions on localizability of the electromagnetic interaction. However, there is no connection between gauge invariance and localizability. Abandoning gauge invariance is equivalent to allowing nonconservation of charge, and Aharonov and Bohm point out that charge is not conserved in their gauge-dependent electromagnetic theory.

There is a nonlocal gauge invariant theory[2] in which charge is conserved. Using the notation of Landau and Lifshitz,[3] the action is

$$S=\frac{i}{16\pi c}\int F_{ik}F_{ik}d\Omega -\frac{i}{c^2}\int A_i(x)g\big((x_j'-x_j)(x_j'-x_j)\big)j_i(x')d\Omega' d\Omega, \quad (1)$$

where

$$\int g(x_jx_j)d\Omega=1 \quad (2)$$

and $g(x_jx_j)\to 0$ as $|x_jx_j|\to\infty$. Equation (1) can be written in the usual form by defining a nonlocal current

$$J_i(x)=\int g\big((x_j'-x_j)(x_j'-x_j)\big)j_i(x')d\Omega'. \quad (3)$$

Then this nonlocal current is conserved

$$\partial J_i(x)/\partial x_i=0 \quad (4)$$

and provided that g is sufficiently well behaved[4] so that Eq. (3) can be solved for j_i, it follows that the current j_i is conserved

$$\partial j_i(x)/\partial x_i=0. \quad (5)$$

The second example is a local gauge-dependent theory obtained by taking as the action

$$S=S_M+if\int A_iA_id\Omega, \quad (6)$$

where

$$S_M=\frac{i}{16\pi c}\int F_{ik}F_{ik}d\Omega-\frac{i}{c^2}\int A_ij_id\Omega. \quad (7)$$

The field equations

$$(4\pi/c)j_i-8\pi fcA_i=\partial F_{ik}/\partial x_k \quad (8)$$

are those used by Lyttleton and Bondi.[5] The current j_i is not conserved,

$$\partial j_i/\partial x_i=2c^2f(\partial A_i/\partial x_i). \quad (9)$$

In this theory, the potential has a physical meaning.

As it is possible to have an electromagnetic theory which is local but not gauge invariant, and a theory which is nonlocal but gauge invariant, there is no relation between gauge invariance and localizability. Aharonov and Bohm[1] point out that for their gauge-dependent theory, the equations for $\mathcal{E}$ and $\mathcal{H}$ have "effective sources" which depend on the potentials and argue that this alters the localizability of the interaction. If the gauge-dependent theory discussed here were interpreted in the same way, it would also appear that the "effective sources" for $\mathcal{E}$ and $\mathcal{H}$ depend on the potentials. However, accepting the viewpoint put forward by Aharonov and Bohm,[6] that the locality of the interaction of electromagnetic field with a charge can only be brought properly into the theory by taking into account that the interaction goes by the intermediary of the potentials, it seems that the gauge-dependent theories of Aharonov and Bohm, and of Lyttleton and Bondi can be best described as local theories.

[1] Y. Aharonov and D. Bohm, Phys. Rev. **130**, 1625 (1963).
[2] H. McManus, Proc. Roy. Soc. (London) **A195**, 323 (1948).
[3] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1951).
[4] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 465.
[5] R. A. Lyttleton and H. Bondi, Proc. Roy. Soc. (London) **A252**, 313 (1959).
[6] Y. Aharonov and D. Bohm, Phys. Rev. **123**, 1511 (1961), see Appendix.